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Dr. Abbas Abdul-Khader Abdullah Al-Shalan
College of Administration and Economics, Al-Muthanna University, Iraq

Correspondence Author:
Dr. Abbas Abdul-Khader Abdullah Al-Shalan
College of Administration and Economics, Al-Muthanna University, Iraq

Formulating the investment portfolio using the entropy measure based on the univariate kernel density function: An analytical study in the Iraqi banking sector

Abbas Abdul-Khader Abdullah Al-Shalan

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Abstract
This study aimed to introduce the entropy measure based on the univariate kernel density function, due to its importance in formulating the returns of the investment portfolio for the banking sector, which is characterized by random returns and does not follow a normal distribution. This makes prediction using traditional measures (such as mean and standard deviation) questionable, as they assume a normal distribution. The research sample consisted of 24 commercial banks within the banking sector for monthly returns in 2024. The study proved that the monthly returns of banks do not follow a normal distribution and demonstrated the ability of the entropy measure to formulate an investment portfolio that aligns with investor expectations and actual returns. It also showed that it can diversify investment portfolios between conservative and risk-taking portfolios. Therefore, the study recommended adopting non-traditional measures such as entropy when predicting price movements and returns and formulating investment portfolios, as such measures avoid unrealistic assumptions that affect the correct selection of stocks.

Keyword: Investment Portfolio, Entropy Measure, Univariate Kernel Density Function

Introduction
The study of investment theory plays a crucial role in real-world investment activities. Traditional investment theory is mainly represented by the portfolio theory proposed by Markowitz, and the basic investment model is the expected return-standard deviation model. However, this traditional model, when diversifying a portfolio, does not fully measure risk, as it usually overlooks the random movement of returns. Hence, this research focuses on the concept of entropy and studies the logic of portfolio diversification, taking into account non-normal return distributions. In portfolio selection theory, risk is understood as the total deviation of returns. Although it has been improved to consider only returns that are below the mean, known as semi-variance, it still does not account for non-normal distributions. Entropy is a measure of stock diversity within a portfolio; the higher the level of entropy, the higher the degree of portfolio diversification. Therefore, entropy was used as an objective function to determine portfolio weights instead of relying on standard deviation in building a portfolio selection model. This research not only studies the concept of entropy but also explores one of its key components, the univariate kernel density estimation (KDE), which is a statistical technique for estimating the probability density function of a random variable. It also analyzes data from the Iraqi banking sector and tests the entropy model so that it can be applied to actual investment activities.

The first chapter: Research methodology
The first requirement: Research methodology
First: Research problem:
The research problem lies in choosing a measure that goes beyond the normal distribution of data, which is an obstacle in traditional measures, and contributes effectively to building an investment portfolio without taking into account the normal distribution of portfolio returns.

It also contributes to overcoming the risks that the portfolio is exposed to through diversification that is consistent with the reality of investment. Therefore, the entropy measure based on single-core probability density was used as an alternative tool to these measures in financial sciences, which has proven its accuracy in other scientific fields. The research attempts to answer the following questions:

1. Does the entropy measure deal with data that does not follow a normal distribution?
2. Can the entropy measure diversify the investment portfolio to avoid risks?
3. Can the entropy measure build an investment portfolio that is consistent with the actual returns of commercial banks instead of predicting from outside the sample or expecting future values?

Second: Importance of research

The importance of research lies in using a non-traditional measure in measuring and diversifying the investment portfolio and analyzing its returns and risks, which is characterized by its accuracy compared to other traditional measures such as variance, semi-variance, or standard deviation. This enhances the performance of the portfolio in the local market (Iraqi banking sector) and avoids risks that deter investors due to weak investment forecasting means. The use of the entropy measure also reduces the required data used in portfolio diversification, such as calculating the correlation coefficient and covariance.

Third: Research objectives

The research aims to build an investment portfolio composed of shares of commercial banks in the Iraqi banking sector that does not follow traditional methods in its formation. The research also aims to overcome the randomness of data that is analyzed when calculating the return and risk of the portfolio, achieving the best trade-off between them. In addition to

1. Recognizing the concept of entropy based on single-core probability density
2. Creating a measure (entropy) that aims to improve the investment portfolio and make it less risky and more diversified.
3. Studying the limitations and solutions of improving portfolios based on entropy and providing guidelines for them.

Forth: Research population and sample

The research was conducted in the Iraqi banking sector as listed in the Iraq Stock Exchange, where (24) banks were selected from the listed banks and other banks were excluded due to the unavailability of their data. The research relied on the opening price, closing price, and percentage change to calculate the returns of commercial banks, as well as the trading value for each bank during (12) months in 2024. These banks were randomly selected only because they provide their data compared to others.

Fifth: Research hypotheses

The research was based on the main hypothesis that (the possibility of entropy measure to diversify the investment portfolio without the need to assume the normal distribution of returns). Some sub-hypotheses emerged from it:

1. The ability of entropy to form an investment portfolio that is consistent with the realistic desire of the investor.
2. The ability of entropy to predict from within the data and not rely on assuming future expected values.

Sixth: Statistical methods used

Some statistical software such as Excel and Spss were used to apply statistical methods such as calculating the average return and standard deviation of commercial banks' returns, in addition to calculating the return and risk of the investment portfolio. The kernel probability density of the data and its numerical integration were also measured to enable us to measure the entropy according to the following formula:

$$H(X) = -\int f(x)\log_2 f(x)\,dx \qquad (1)$$

Where: H(X): Entropy of the continuous random variable, f(x): Kernel probability density function, log_2 f(x): Logarithm of the probability density, dx: Differential value.

The second chapter: Theoretical framework.

The first requirement: Investment portfolio.

First: Concept of investment portfolio

Modern portfolio theory studies the interrelationship between expected return and risk and discusses how to minimize risk and maximize return by setting appropriate investment objectives for the portfolio, determining the optimal proportion of securities within the specified objectives, and changing portfolio components in a timely manner in response to complex changes in the financial market and knowing the proportion of each part or type of securities. It also focused on identifying investors' constraints, investment needs, recurring income, and capital size to meet investors' objectives. Its development has significantly changed traditional investment management practices, making modern investment management increasingly oriented towards organization, science, and integration (Shirin, T., 2023: 25-26) [12].

Harry Markowitz, the American economist, was the first to propose the investment portfolio in 1952, as his investment model measured by expected return and standard deviation is considered one of the most influential and widely used portfolio models, and his idea formed achieving a suitable balance between income and risk. Tobin (1969) expanded the scope of portfolio theory to include analysis of all real and financial assets, thus forming the theory of asset selection. In light of this, Zuo and Tang (2004) identified the main components of portfolio management, which are investment policy, investment analysis, ass*et al*location, portfolio adjustment, and portfolio performance (Yin, Dechao, 2019: 1214) [17].

Markowitz's hypothesis was based on the idea that investors diversify their investments across a set of securities to form an investment portfolio rather than investing in a single asset. Markowitz clarified that the primary concern of investors should be building an optimal asset portfolio that minimizes risk at a specified level of expected return or maximizes return at a specified level of risk, relying on mean-variance to select securities to achieve returns and risk effectively. Markowitz addressed a set of fundamental

challenges that have long hindered investment in securities, including:

1. **Diversification of investments:** Diversifying investments across a portfolio of securities can significantly reduce investment risk.
2. **Determining the efficient frontier:** Determining the efficient frontier under a fixed set of securities, and that differences in investment proportions lead to an infinite number of portfolios.
3. **Risk and return patterns:** Each portfolio has different patterns of risk and return, collectively known as the feasible set or possible set
4. **Optimal portfolio selection:** The selection of an optimal investment portfolio along the efficient frontier is influenced by the investor's risk preferences, which are represented by a set of indifference curves.

Accordingly, an investment portfolio is a set of investments owned by the same individual or institution, including securities and financial assets such as stocks, bonds, and mutual funds, and typically, portfolio investments are diversified, and each asset carries varying levels of risk However, investment returns are not fixed but are closely linked to risk, as any investment behavior carries risk, and although some financial products achieve high investment returns, they are in fact accompanied by high investment risk, high returns correspond to high risk, which is the general rule for all mature markets, yet it can achieve expected benefits in a complex and volatile financial market by efficiently avoiding investment risk (Yin, Dechao, 2019: 1215) [17], as portfolio theory is based on a set of assumptions, including (Kapoor, N., 2014: 1366) [8]

1. **Investors avoid risk:** Investors avoid risk, meaning that instead of investing their entire wealth in a single asset, they maintain a well-diversified portfolio with the goal of providing maximum return at a certain level of risk, or ensuring a minimum level of risk at a certain level of return.
2. **Securities returns are normally distributed:** Securities returns are normally distributed, meaning that the mean and variance or standard deviation are the basis for portfolio decisions.
3. **Determining investment portfolio objectives:** Determining investment portfolio objectives and selecting the securities to be included.
4. **Needs analysis:** Investors analyze income needs, liquidity, capital safety, time horizon, tax considerations, and temperament.
5. **Securities selection:** Securities selection is based on risk and return analysis, which includes market returns and dividend distributions.
6. **Ass*et al*location:** Ass*et al*location, meaning selecting the portfolio that meets the investor's requirements.

An investor can maintain the asset class distribution ratio and keep the securities holdings in place during the specified holding period, or the investor continuously evaluates the risks and returns of securities within asset classes and adjusts them accordingly, as diversification reduces risk. According to the simple diversification concept, researchers say that if 15 stocks are added to the investor's portfolio, unsystematic risk can be reduced to zero, but at the same time, if the number exceeds 15 stocks, no additional risk reduction can be achieved. However, diversification does not reduce systematic or non-diversifiable risk. According to Markowitz's concept, he emphasized the importance of measuring the correlation coefficient between the returns of different securities in building a stock portfolio (Baker, H. & Filbeck, G., 2013: 1-2) [1]. Assets can interact in different ways with the same market forces, and assets that move in opposite directions in response to the same economic changes or market forces are described as having low or negative correlation. When holding these assets together in a diversified portfolio, losses in one part of the portfolio are likely to be offset by gains in another part. Often, the prospect of rising inflation is detrimental to the bond market, and the available income from bonds is offset, thus its value decreases when inflation rises. In contrast, stock markets tend to adapt better to rising inflation, partly due to companies' ability to raise prices to counter it, which is reflected in their stock prices (Xiaoyu Li, 2024: 1044).

Markowitz has faced criticism because his theory only considers rational investors, despite behavioral economics showing that individuals often make irrational decisions in many areas of their lives, which may be due to emotional factors, forced decision-making, or differences in information flow (Gábor & László, 2019: 7). Nevertheless, these models are typically based on the assumption of a "perfectly rational man" with a primary focus on asset returns and transaction costs to analyze portfolio problems. Based on this, the standard deviation measure remains dominant, with 83% of asset management companies using it to improve their portfolio performance (Xiaoyu Li, 2024: 1045) [7]. However, research has pointed out some drawbacks of this theory, including.

1. **The theory relies on the first moments of the sample:** The theory relies on the mean and standard deviation of the sample, which may not be sufficient to accurately represent risk and return.
2. **Focus on a limited number of assets:** The model often focuses on a limited number of assets, leading to a less diversified investment portfolio and potentially increasing risk.
3. **Uncertainty in data:** The theory does not give sufficient attention to uncertainty in data and often relies on inaccurate estimation models such as mean and variance.
4. **Model sensitivity to inputs:** Differences in input data significantly affect the portfolio, and some new observations may completely change the portfolio.
5. **Non-normal distribution of returns:** Evidence shows that almost all asset classes and investment portfolios achieve returns that are not normally distributed, and the first two moments are generally not sufficient to explain investment portfolios in the case of non-normal return distributions.

The portfolio selection problem can be determined by considering a set of n assets, each with actual returns R_1, ..., R_n, by allocating weights w_1, ..., w_n to the capital invested in each asset. Since Markowitz aims to maximize portfolio return and minimize risk, represented by the standard

deviation of portfolio returns, this can be achieved through the following formula: (Wen, Z., 2023: 100)

$$R_p = \sum_{i-1}^{n} w_i R_i \quad (2)$$

$$\text{subject to } \; w_1 + \cdots + w_n = 1, \;\; w_i \geq 0 \; \forall \, i,$$

Where R_p is the weighted return of the portfolio, w_i is the weight of the security in the portfolio, R_i is the actual return of the security within the portfolio, ∀i denotes the universal quantifier. Portfolio risk is calculated using the following formula: (Wen, Z., 2023: 100)

$$\sigma P = \sqrt{\sum_{i=1}^{N} \mathrm{Wi}^2 \, \sigma i^2 \; + 2 \sum_{i=1}^{N-1} \; i \neq j \;\; \sum_{j=1}^{N} \mathrm{Wi} \, \mathrm{Wj} \, \mathrm{Cij} \, \sigma i \, \sigma j}$$

σP is the portfolio risk measured by standard deviation, w_i is the weight of the security in the portfolio and σ is the standard deviation of the security within the portfolio, n is the number of securities in the portfolio, Cij is the correlation coefficient between each pair of securities ij in the portfolio, and the correlation coefficient is calculated according to the following formula: (Yu, J. N., 2020: 3)

$$p_{ij} = \frac{COV_{ij}}{\sigma_i \times \sigma_j} \quad (4)$$

Where COV_{ij} is the covariance between security i and security j, σ_i is the standard deviation of security i, and σ_j is the standard deviation of security j. To calculate the covariance between each pair of securities ij, the following formula can be used.

$$COV_{ij} = \frac{\sum(r_i - \bar{r_i})(r_j - \bar{r_j})}{n} \quad (5)$$

To calculate the weight of the security w from the invested capital within the portfolio, we use the following formula:

$$\mathrm{w_i} = \frac{\sigma_j}{\sigma_i + \sigma_j},$$

$$\mathrm{w_j} = \frac{\sigma_i}{\sigma_j + \sigma_i} \quad (6)$$

Second requirement - Entropy measure based on kernel density function

First, the concept of entropy

Clausius (1854) first used the term "entropy" in the context of thermodynamics to estimate the ratio of heat transferred from one body to another in an isolated system, which increases with increasing heat levels, and was named Clausius entropy. Boltzmann (1872) linked entropy to the value of direction and speed of particles by corresponding coordinates at a specific point in mathematical space. Thus, the probability of arranging these particles is directly proportional to the number of states they create, and some arrangements are more likely to occur than others, as the system spends more time in the most probable arrangements (Chortane, S. & Naoui, K., 2022: 47) [3].

Once the concept of entropy was introduced, it gained an important theoretical position in the field of thermodynamics and statistical mechanics. It came to indicate the probability of the thermal state of matter in thermodynamics, as well as indicate uncertainty about a situation or problem in mathematics, and the ability of a source to send information in information theory, which was introduced by Claude Shannon in 1948 to solve the problem of quantitative measurement of information, and describe uncertainty in the source. The concept of information entropy paved the way for quantitative research in the field of financial research in financial mathematics. With the continuous development of investment theory, information entropy gradually entered the financial field, which helped people to invest more efficiently, until it became a measure of risk in investment that matches the probability of uncertainty or chaos (Kapur, J.N., 1982: 698-700) [9].

Many studies have shown that entropy analysis measures meaningful information that is not available through traditional statistical techniques such as analysis of variance or correlation, and that it is non-parametric, making it a better statistical measure of risk than standard deviation. Therefore, entropy has been used as a measure of risk. The smaller the value of entropy, the less uncertainty the stock return contains, and thus the stock return is more stable and secure. On the other hand, if the entropy value of the portfolio increases, it indicates that the portfolio is more diversified and its stocks do not move with a uniform probability distribution. Moreover, entropy does not rely on symmetrical distributions of security returns and can be calculated from non-metric data. Based on Shannon's definition, entropy is the degree of uncertainty or disorder in a portfolio or security, and it can also be defined as the probability that measures disturbances. It represents the average uncertainty of an investor about the returns of a project. Being distribution-free, it is not affected by errors resulting from fitting the distribution of returns to a specific distribution. Macaulay (2003) argues that entropy is capable of capturing the complexity of systems without the need for unrealistic assumptions that can affect the results obtained (Wang, *et al*, 2022: 2-3) [13].

Since entropy characterizes uncertainty or measures the dispersion of a random variable and indicates chaos, in our current research, it characterizes uncertainty and dispersion regarding stock returns and portfolio. Unlike the modern Markowitz portfolio, which assumes that returns are constant and normally distributed, but this is not the case in reality. The more difficult it is to predict an event, the more information it contains, and the variable is completely disturbed or dispersed when its entropy is at its maximum in a time series (Chortane, S. & Naoui, K., 2022: 51) [3]. To illustrate the susceptibility of a non-regular variable to

influence, the entropy of that variable is used to clarify the risks of stock returns and investment portfolios. Entropy has been used as an alternative to estimating the difference in risk (Wang, *et al*, 2022: 2-3) [13]. The key differences between Markowitz's modern portfolio theory and the entropy-based portfolio can be identified, as shown in Table (1).

Table 1: Modern Portfolio vs. Entropy Portfolio

Modern Portfolio	Entropy Portfolio
Provides a theoretical basis for diversification and an analytical framework for effective diversification, assuming normal data distribution.	Measures density spread regardless of period or trend, not linked to normal or random data distribution.
Difficulty in estimating covariance matrix and expected returns, historical returns may not be useful.	Provides a natural probabilistic interpretation of daily returns, which typically change continuously.
When the number of securities is large, the required estimator for basic inputs is too large, limiting the use of the Markowitz model.	Entropy may be linked to high-order moments of the distribution, providing a more accurate description of probability.
Unreliability of the solution due to data errors, and significant differences in optimal solutions when adjusting inputs or considering investor opinions.	Handles non-normal or asymmetrical returns, and in the absence of known probability distribution, entropy serves as an alternative measure of uncertainty.
Inaccurate data entry due to statistical estimates can lead to over-investment in some asset classes and under-investment in others.	Handles large data, with increasing accuracy over longer time series.
Standard deviation cannot accurately represent risk, as the sign of the error does not affect volatility.	Explanatory power is higher for entropy-based risk measures compared to standard deviation.
Many asset returns are non-symmetrical, skewed, or kurtotic.	Can be used as a complement to mean-variance portfolio model, automatically determining the degree of inaccuracy in estimating mean and covariance matrix.
Adjustments in asset proportions can lead to unnecessary transaction costs.	Lower entropy values indicate more stable stock returns and less uncertainty, while high entropy in portfolio investment indicates diversification, with stocks moving inconsistently and chaotically.

Source: Adapted by the researcher based on several sources

According to the entropy measure, the entropy value is represented in a probability function and must meet the following properties: (Maasoumi & Racine, 2002) [10]

1. The entropy value of two events is higher than the value of each event individually.
2. If the two events are independent, the entropy value will be the sum of the two events.
3. The entropy value in any event is non-negative.

And the only mathematical function that meets all the mentioned properties for a continuous random variable is:

$$H(X) = -\int f(x)\log_2 f(x)\,dx \qquad (7)$$

Where (X) is a discrete random variable for a limited time range, such as (X_1, X_2,… X_n), and H(X) represents the entropy of the variable X, and $\int f(x)$ is the probability density function of the continuous variable, calculated according to the kernel density. The probability density of the continuous variable cannot be a single integer, because it is a mathematical function defined on a set of values. This means that the probability density can take any non-negative value, but the integral over all possible values must equal one when multiplied by the differential value dx of the continuous variable. The logarithm appearing in the above equation is the entropy scaling factor [1]. If the variable is discrete, entropy can be measured using the following formula:

$$H(X) = -\sum_{x_i \in X} p(x_i)\log_2 p(x_i) \qquad (8)$$

Where $p(x_i)$ is the probability of occurrence of (X) assuming the value (x_i), The probability must be greater than or equal to zero $p(x_i) \geq 0$, The sum of probabilities equals one $\Sigma_{x_i \in X} p(x_i) = 1$, Entropy is determined by a logarithmic base consistent with the unit of measurement used ،(H) denotes the amount of entropy, Entropy reaches zero as a minimum when the variable (X) is constant and is characterized by a completely concentrated probability distribution $p(x) = 1$. Conversely, the amount of entropy is at its maximum when the probability distributions are uniform, meaning all states (x_i), are equally likely to occur.

Secondly: Kernel Density Estimation (KDE)

Given that the concept of entropy is related to uncertainty about the outcome of a random variable, which is a very common phenomenon in financial markets, the only satisfactory description of uncertainty is probability.

[1] Base 2 logarithm is used to represent the minimum number of bits required to store a message without losing information, as in information theory

Therefore, any measure of uncertainty must be in the form of probability. When calculating portfolio diversity based on entropy, using probability as an objective function to determine portfolio weights is enabled. Instead, the portfolio is more informed in the event of an unlikely occurrence. Therefore, when measuring entropy, it is necessary to know the formula and implications of the expressions for the measured variables (Rödder, W., *et al*, 2009: 5-6).[11]

The history of using kernel functions to estimate the probability density function (KDE) dates back to Rosenblatt (1956), who proposed kernel estimation for continuous univariate probability density functions and derived the infinite mean square error based on symmetric kernel functions. Parzen (1962) extended kernel estimation to multivariate probability density functions and developed a general formula for the optimal bandwidth, which is a smoothing parameter that controls the balance between bias and variance in estimation. Since then, many researchers have developed various methods for selecting the bandwidth, such as direct plug-in methods, cross-validation methods, and approximate methods (Silverman, 1986). Kernel density estimation has been widely used in various fields due to its ability to describe the shape and properties of data without imposing any parametric assumptions (Belhaj, E. I., 2024: 143) [2].

In statistics, univariate kernel density estimation (KDE) is a non-parametric method for estimating the probability density function (f(x)) of a random variable X. It is a fundamental solution to the problem of data smoothing, where inferences are drawn about the statistical population based on a limited data sample. This technique is widely used in various inference procedures, such as signal processing, data mining, econometrics, and estimating modes and inflection points in density distributions, which apply smoothing techniques and calculate bandwidth values for the derivative of degree "r" of the probability density function f(x) for univariate data (Guidoum, A. C., 2020: 1-2).

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right) \quad (9)$$

Where $\hat{f}(x)$ is the estimated probability density, (x) is the point at which the probability density is estimated, x_i are the individual data values, h is the kernel bandwidth, and K is the Gaussian kernel. To calculate the density, we will use SPSS software to calculate the kernel probability density.

Among all probability distribution functions, the probability density function best shows how the total probability (100%) is distributed on the horizontal axis, i.e., on the values of the random variable. Its shape depends on the choice of the number or size of classes into which the sample range is divided, and on the choice of the starting point. To solve this problem, researchers have proposed several formulas, most of which depend only on the sample size, while some include other characteristics of the sample such as standard deviation, interquartile range, or skewness. Regardless of the method of choosing classes, estimating the probability density function using the kernel does not suffer from these drawbacks, as it provides (in most practical applications) a smooth empirical probability density function that depends on the locations of each element in the sample. This estimate seems to represent the true probability density function of the continuous variable better. Kernel estimation technique is not entirely new, it was invented by Rosenblatt and Parzen more than half a century ago. With the development of computer technology, this method has developed significantly (Węglarczyk, S., 2018: 1) [14]. It does not require assumptions about the specific shape of the distribution, and can provide accurate estimates of complex or multimodal distributions. It can be used to create probability density plots that help understand the distribution of data. In general, KDE is a powerful and flexible tool for data analysis and understanding its probability distributions.

The third section - The practical side of the research

This part of the research adopts the calculation of mathematical equations and statistical procedures to apply the entropy measure based on the univariate kernel density for a group of banks in the banking sector listed in the Iraq Stock Exchange with the aim of testing and discussing the research hypotheses. This section measures the entropy of the returns of commercial banks, which allows investors to form their investment portfolio. Low entropy between bank returns indicates stability and low uncertainty in their realization, and vice versa, banks with high entropy indicate large fluctuations in their returns. Within the investment portfolio, stock selection is based on high entropy, which indicates diversification and dissimilarity of selected stocks in their return movements. Table (2) shows the monthly returns for a sample of commercial banks in the banking sector (24 banks) during the year 2024, calculated based on the percentage change in opening and closing prices

Table 2: Commercial Banks' Returns

Bank Name Return	Dec	Nov	Oct	Sep	Aug	Jul	Jun	May	Apr	Mar	Feb	Jan
Iraqi Commercial Bank	2.86	0	-6.25	17.07	0	2.5	-9.09	12.82	1.3	-13.48	1.14	22.22
Baghdad Islamic Bank	-6.42	1.48	31.49	6.57	-1.37	-4.56	-0.32	-26.67	37.7	-15.51	4.03	-0.86
Iraqi Islamic Bank	6.15	-5.13	15.56	25	-5.26	6.54	0	-13.71	39.33	-9.18	8.89	30.43
Middle East Bank	9.09	-12.5	14.29	-22.22	-10	0	-9.09	0	10	-9.09	0	-8.33
Iraqi Investment Bank	16.22	-13.33	-3.23	-6.06	-2.94	-2.86	-7.89	5.56	5.88	-8.11	-17.78	4.65
National Bank of Iraq	1.67	11.11	25.19	16.67	-9.88	-7.66	-2.49	-7.87	-12.86	-0.57	15.79	25.1
Sumer Commercial Bank	-13.33	-11.11	0	12.5	-20	11.11	-10	0	0	-9.09	-8.33	-7.69
Economy Bank	0	-3.85	4	19.05	-4.55	-8.33	-4	25	0	0	-13.04	15
Gulf Commercial Bank	0	0	-3.13	10.34	-14.71	6.25	-21.95	141.18	6.25	-11.11	-5.26	5.56

Mosul Investment Bank	-4.76	0	-5.88	-5.56	0	-10	5.26	5.56	-5.26	-9.52	0	5
Union Bank of Iraq	-3.33	0	-20	-19.35	40.91	0	0	10	-31.03	0	0	0
Ashur Bank	-10.64	-10	0	11.11	-3.57	-3.45	0	-3.33	0	-6.25	-20	-4.76
Kurdistan Bank	0	0	-10	0	-20	25	11.11	5.88	18.06	-15.29	0	0
Al-Mansour Bank	10.24	2.63	10.47	28.36	-2.9	5.34	-0.76	-1.49	3.88	1.57	-26.59	23.57
United Investment Bank	0	-28.57	16.67	-14.29	-12.5	0	0	-11.11	12.5	0	0	0
Elaf Bank	25	0	0	15	-9.09	0	-6.38	-14.55	0	0	0	0
National Islamic Bank	0	0	0	0	0	-8.06	0	0	24	0	0	0
Al-Taif Islamic Bank	23.19	33.33	-3.85	0	-8.24	23.19	-1.43	0	-6.67	7.14	-19.54	2.35
Iraqi Credit Bank	14.49	3.66	-6.82	10	-9.09	-3.3	-3.19	-11.32	15.22	-9.8	-8.11	40.51
Iraq Gateway Bank	0	89.8	0	-10.91	-25.68	0	0	0	13.85	0	0	-48
Al-Ataa Islamic Bank	0	-8.33	-7.69	8.33	-14.29	7.69	-13.33	25	0	-7.69	8.33	-7.69
International Islamic Bank	1.96	16	-26.47	3.03	-5.71	-2.78	-7.69	-2.5	-50	-60	-9.09	-15.38
Babylon Bank	0	0	0	0	-14.29	0	-12.5	14.29	0	-12.5	14.29	-12.5
Noor Al-Iraq Bank	0	35.56	0	-10	-28.57	-33.33	0	0	0	0	-8.7	-14.18

The table above shows the monthly returns of commercial banks with different results, where some banks achieved profits while others incurred losses. Table (2) shows that the highest rate of return achieved by commercial banks was by Gulf Commercial Bank, which reached (141.18%), while its loss rate reached (-21.95%). The table also shows that the lowest rate of return for banks was for the International Islamic Bank, which reached (-60%), while the same bank achieved a profit rate of (16%). This indicates fluctuations in the movement of returns and raises doubts about their normal distribution, making it necessary to test their normal distribution to determine the appropriate mathematical measures to calculate the return and risk of the portfolio, whether traditional measures that rely on normal distribution or the entropy measure that assumes no distribution.

Second: Testing the Normal Distribution of Data

To verify the distribution of commercial banks' returns data, a skewness and kurtosis test was conducted. The results indicate a variation in skewness and kurtosis values from the expected values for a normal distribution (zero and three, respectively). This means that the return data does not follow a normal distribution, making traditional measures unsuitable for measuring it. Instead, it becomes necessary to use measures that do not assume any specific distribution of data, such as the entropy measure. This is shown in Table (3).

Table 3: Statistical Description of Commercial Banks

Bank Name	Months	Average Annual Return	Standard Deviation of Return	Skewness		Kurtosis	
				Statistics	Standard Error	Statistics	Standard Error
Iraqi Commercial Bank	12	2.591	9.957	.818	.637	-.465-	1.232
Baghdad Islamic Bank	12	2.130	16.923	.791	.637	1.030	1.232
Iraqi Islamic Bank	12	8.218	15.852	.601	.637	-.563-	1.232
Middle East Bank	12	-3.154	10.215	.080	.637	-.500-	1.232
Iraqi Investment Bank	12	-2.491	8.952	.373	.637	.189	1.232
National Bank of Iraq	12	4.517	13.113	.367	.637	-1.414-	1.232
Sumer Commercial Bank	12	-4.662	9.270	.533	.637	-.192-	1.232
Economy Bank	12	2.440	10.986	.880	.637	-.064-	1.232
Gulf Commercial Bank	12	9.452	40.758	3.146	.637	10.489	1.232
Mosul Investment Bank	12	-2.097	5.302	.118	.637	-1.285-	1.232
Union Bank of Iraq	12	-1.900	17.026	.858	.637	2.740	1.232
Ashur Bank	12	-4.241	7.178	-.137-	.637	2.000	1.232
Kurdistan Bank	12	1.230	12.332	.192	.637	-.008-	1.232
Al-Mansour Bank	12	4.527	13.228	-.418-	.637	2.018	1.232
United Investment Bank	12	-3.108	11.613	-.460-	.637	.777	1.232
Elaf Bank	12	0.832	9.903	1.180	.637	2.161	1.232
National Islamic Bank	12	1.328	7.187	2.786	.637	9.467	1.232
Al-Taif Islamic Bank	12	4.123	14.578	.667	.637	-.080-	1.232
Iraqi Credit Bank	12	2.688	14.576	1.522	.637	2.419	1.232
Iraq Gateway Bank	12	1.588	30.698	1.801	.637	5.986	1.232
Al-Ataa Islamic Bank	12	-0.806	10.969	.980	.637	.816	1.232
International Islamic Bank	12	-13.219	21.209	-1.191-	.637	.896	1.232
Babylon Bank	12	-1.934	9.294	.377	.637	-.495-	1.232
Noor Iraq Islamic Bank	12	-4.935	16.523	.638	.637	2.474	1.232
Overall Mean	12	0.130	14.068				

As the above table shows, the Gulf Commercial Bank achieved the highest average return of 9.452, which is higher than the general average of 0.130, with a standard deviation of 40.758, which is also higher than the general standard deviation of 14.068. This indicates that the bank achieves high returns but is also exposed to higher risks due

to the significant fluctuation in its returns. In terms of return, the Gulf Commercial Bank is acceptable for forming an investment portfolio, but in terms of standard deviation, it cannot be accepted due to the high risks. The same applies to Al-Tayf Islamic Bank, which achieved a return of 4.123, which is higher than the general average, but its standard deviation was 14.578, which is higher than the general standard deviation, indicating higher risks. Therefore, it is necessary to adopt measures that take into account the significant fluctuation in the returns of commercial banks, such as the entropy measure, when forming and selecting stocks for an investment portfolio.

Measuring the entropy of commercial banks' returns requires knowledge of the probability density function f(x) calculated according to the kernel density estimation. Therefore, we need to transform the average annual return into a probability distribution using integration and differential values, which requires knowledge of the probability density function f(x) of the random variable X, such as kernel density estimation (KDE) using statistical software SPSS to transform the data into a kernel probability density function and then calculate the entropy, as shown in table (4).

Table 4: Measuring the Entropy of Commercial Banks

Bank Name	Probability Density	Differential	(dx)	Logarithm	Entropy	Expected Entropy [2]
Iraqi Commercial Bank	0.0053	10.230	0.0833	-7.55979	0.03414	0.04167
Baghdad Islamic Bank	0.0071	12.560	0.0833	-7.13797	0.05302	
Iraqi Islamic Bank	0.0085	15.670	0.0833	-6.87832	0.07632	
Middle East Bank	0.0049	8.920	0.0833	-7.673	0.02794	
Iraqi Investment Bank	0.0063	11.450	0.0833	-7.31043	0.04393	
National Bank of Iraq	0.0081	14.230	0.0833	-6.94786	0.06671	
Sumer Commercial Bank	0.0056	9.560	0.0833	-7.48036	0.03336	
Economy Bank	0.0065	12.190	0.0833	-7.26534	0.04795	
Gulf Commercial Bank	0.0073	10.980	0.0833	-7.09789	0.04739	
Mosul Investment Bank	0.0042	8.450	0.0833	-7.89539	0.02334	
Union Bank of Iraq	0.0051	11.920	0.0833	-7.61529	0.03856	
Ashur Bank	0.0045	9.230	0.0833	-7.79586	0.02697	
Kurdistan Bank	0.0039	8.120	0.0833	-8.00231	0.02111	
Al-Mansour Bank	0.0078	14.560	0.0833	-7.00231	0.06624	
United Investment Bank	0.0048	10.450	0.0833	-7.70275	0.03218	
Elaf Bank	0.0062	12.890	0.0833	-7.33352	0.04882	
National Islamic Bank	0.0035	8.980	0.0833	-8.15843	0.02136	
Al-Taif Islamic Bank	0.0089	15.340	0.0833	-6.81198	0.07747	
Iraqi Credit Bank	0.0075	13.450	0.0833	-7.05889	0.05932	
Iraq Gateway Bank	0.0059	11.230	0.0833	-7.40507	0.04087	
Al-Ataa Islamic Bank	0.0068	12.560	0.0833	-7.20025	0.05123	
International Islamic Bank	0.0082	14.980	0.0833	-6.93016	0.07091	
Babylon Bank	0.0041	9.890	0.0833	-7.93016	0.02679	
Noor Iraq Islamic Bank	0.0061	11.780	0.0833	-7.35698	0.04404	

The table above shows the entropy values for the returns of commercial banks and indicates the disturbance in the movement of return rates or their stability. To distinguish between them, they are compared to the hypothetical mean of entropy for the banking sector, which is calculated to be (0.04167). The highest entropy value was for Al-Tayf Islamic Bank, which was (0.07747), indicating disturbances and high dispersion in the bank's returns and their instability. On the other hand, the lowest entropy value was for Kurdistan Bank, which was (0.02111), indicating that the bank is characterized by low uncertainty in predicting its returns and that they are more stable compared to other returns. Based on the hypothetical mean, an investment portfolio with the most stable returns will be formed, as shown in table (5).

Table 5: The Portfolio with Low Entropy

Bank Name:	Entropy	Trading Value	Standard Deviation	Average Return	Stock Weight
Commercial Bank	0.034143	5.52E+09	10.3999	2.5908	0.1974
Middle East Bank	0.027936	8.5E+08	10.66886	-3.1542	0.0304
Sumer Commercial Bank	0.033359	5.1E+09	9.68214	-4.6617	0.1823
Mosul Investment Bank	0.023341	1.61E+09	5.53725	-2.0967	0.0575
Iraqi Union Bank	0.038564	35841821	17.78324	-1.9000	0.0013
Ashur International Bank	0.026973	1.55E+09	7.49716	-4.2408	0.0553
Kurdistan Bank	0.02111	97603414	12.87986	1.2300	0.0035
United Bank	0.032185	5.1E+08	12.12934	-3.1083	0.0182
National Islamic Bank	0.02136	1.25E+10	7.5063	1.3283	0.4483

2-The hypothetical mean equals: $H(X) = - P(1/2) \log_2 P(1/2) \div n$
Where P(1/2) represents the probability of occurrence n, n represents the number of investments

Trans Iraq Bank	0.04087	1543540	32.06255	1.5883	0.0001
Babylon Bank	0.026786	1.58E+08	9.70693	-1.9342	0.0056
Portfolio Correlation	0.008				
Portfolio Return	-0.25963				
Portfolio Risk (Standard Deviation)	4.39				

The table above shows the statistical indicators for the portfolio with low entropy. The correlation coefficient between the banks' returns was (0.008), indicating a weak correlation between the banks. The average return on the investment portfolio was (-0.259), indicating that the portfolio lost part of its investment capital. The portfolio risk was (4.39), calculated by standard deviation, which is less than the risk of each bank individually, indicating portfolio diversification. To form the portfolio with high entropy, as shown in table (6).

Table 6: The Portfolio with High Entropy

Bank Name	Entropy	Trading Value	Standard Deviation	Average Return	Stock Weight
Baghdad Bank	0.053023	220,776,977,299	17.67546	2.1300	0.7864
Iraqi Islamic Bank	0.076316	4,016,367,283	16.55652	8.2183	0.0143
Iraqi Investment Bank	0.043927	3,873,527,324	9.34962	-2.4908	0.0138
National Bank of Iraq	0.066709	20,023,995,765	13.69575	4.5167	0.0713
Economy Bank	0.047953	1,968,366,607	11.47493	2.4400	0.0070
Gulf Commercial Bank	0.047391	3,316,127,305	42.57061	9.4517	0.0118
Al-Mansour Bank	0.066243	17,111,110,755	13.81571	4.5267	0.0609
Elaf Islamic Bank	0.04882	3,811,993	10.34292	0.8317	0.0000
Al-Tayf Islamic Bank	0.07747	512,721,762	15.22578	4.1225	0.0018
Iraqi Credit Bank	0.059315	4,177,347,656	15.22424	2.6875	0.0149
Al-Atta Islamic Bank	0.051226	4,656,656,000	11.45668	-0.8058	0.0166
International Islamic Bank	0.070911	314,753,385	22.15158	-13.2192	0.0011
Noor Iraq Islamic Bank	0.044037	3,048,500	17.25783	-4.9350	0.0000
Correlation Coefficient	0.128				
Portfolio Return	2.504226				
Portfolio Risk	22.25				

The table above shows the statistical indicators for the portfolio with high entropy, indicating a high level of disorder in the movement of returns, which provides better diversification. The returns on the stocks do not move in a consistent manner. The portfolio return was (2.504), indicating a profit for the portfolio. The portfolio risk, calculated by standard deviation, was (22.25), which is a high risk but is consistent with the achieved return and the risk appetite of investors seeking to take risks. Therefore, the investor has the choice of which stocks to include in their portfolio, now that the movement of returns on commercial banks and the extent of their consistency or conflict has become clear, and which is not based on the assumption of normal distribution of returns, but rather is closer to the reality of bank return movements.

In light of the foregoing, the hypothesis that (the entropy measure can diversify the portfolio without the need for normal distribution assumptions, as in traditional measures) can be proven. The tests have shown that the returns on commercial banks are not normally distributed, and it is better to avoid using traditional measures (mean return and standard deviation in stock selection). The mean return relies on returns that are greater than the general average, assuming all returns have a normal distribution. The standard deviation in stock selection also relies on stocks with lower deviation, i.e., those with less dispersion from their mean, assuming all returns have a normal distribution. A realistic investor does not want to neutralize data, interpret it, and make it conform to the assumption of normal distribution, which is not consistent with the movement of securities in the markets. The study also proved the first sub-hypothesis that (entropy can form an investment portfolio that is consistent with the realistic desires of the investor). Entropy was able to distinguish between banks with low entropy, characterized by stable returns, and banks with high entropy, characterized by fluctuating returns. The former is consistent with the desires of cautious investors, and the latter is consistent with the desires of risk-taking investors. The study also proved the second sub-hypothesis that (entropy can predict from within the data, without relying on assumptions of expected future values). Entropy relied on tracking the actual distribution of returns itself to distinguish between fluctuations in returns, without resorting to using expected future values, which are usually based on a set of assumptions.

Conclusion and Recommendations

Firstly Conclusion

1. Stock returns do not follow a normal distribution, but rather a random movement, which affects the forecasting process when using traditional methods.
2. The entropy measure depends on the probability distribution of returns, and it is necessary to distinguish between the probability distributions of discrete and continuous variables.
3. Commercial bank returns are a continuous variable that ranges between positive and negative, so it is necessary to use the kernel probability density function to deal with this variation and negative returns.
4. When calculating entropy for individual commercial banks, low entropy indicates stable returns and less uncertainty in predicting their movement.
5. When calculating entropy at the portfolio level, low entropy indicates less diversification, while high

entropy indicates better diversification.
6. Entropy depends on actual data and its distribution, and the probability of its occurrence, and does not rely on forecasting future data.

Secondly. Recommendations

1. The research recommends adopting non-traditional measures such as entropy because it does not depend on data distribution.
2. The research recommends that investors use entropy when forming investment portfolios.
3. The research recommends adopting the entropy measure because it does not require large data or statistical indicators.
4. The research recommends that commercial banks adopt the calculated entropy amount to adjust the path of their returns and reduce chaos.
5. The research recommends adopting and developing such measures in the Iraq Stock Exchange.

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